corresponds to the minimal power transmitted in the fade-out range. The minimal power transmitted is not exactly the value of interest, though. The exact goal consists in suppressing the maximum value of the power density spectrum $S_s(e^{j\theta})$ within the fade-out range below a certain value. Of course, a minimum/maximum criterion may be formulated therefor, but this criterion is more difficult to manipulate mathematically.

This is the reason why the function used is not a rectangular window function $W(e^{j\theta})$, but rather a relatively smooth weighting function. The transitions from 1 to zero and reverse are thereby carried out with linear gradients. This choice does not correspond to the minimum/maximum solution, neither does it to the minimization of the power transmitted within the fade-out range, but simulations proved that this choice yields better results than the minimization of the transmitting power within certain frequency bands.

The coefficients $C[r]$, r = 0, 1, ..., R-1 may now be written down as the solution to the optimization problem

$$\begin{pmatrix} C[0] \\ C[1] \\ \vdots \\ C[R-1] \end{pmatrix} = \arg \min_{\substack{C[0] \\ C[1] \\ \vdots \\ C[R-1]}} \psi_1(C[0], C[1], \ldots, C[R-1]) \,. \tag{122}$$

Because (121) is a square function in the coefficients $C[r]$, the equation (121) has one single minimum. This minimum is described by the equations

$$\frac{\partial}{\partial [C[r]]_{ui}} \psi_1(C[0], C[1], \ldots, C[R-1]) = 0 \qquad \text{for} \qquad \begin{array}{l} r = 0, 1, \ldots, R-1 \\ u = u_1, u_2, \ldots, u_U \\ r = i_1, i_2, \ldots, i_I \end{array} \tag{123}$$

that may be solved by linear systems of equations that are independent of U. Each value of u corresponds to a linear system of equations with RI unknowns. The system of equations that pertains to a fixed value of $u_x \in U$, describes the coefficients that are contained in the x^{th} line of all the R matrices $\mathbf{C}(r)$, r = 0, 1, 2, ..., R - 1. If the x^{th} line of the matrix $\mathbf{C}(r)$ is designated as $\mathbf{c}^T_{ux}(r)$ (compare equation (112)), the vector of the unknowns can be written down as $\mathbf{c}^T_{ux} = (\mathbf{c}^T_{ux}(0)\mathbf{c}^T_{ux}(1) \ldots \mathbf{c}^T_{ux}(R-1))$. At first sight it seems strange that the system of equations described by equation (123) splits into U-independent systems of equation. The coefficients contained in the x^{th} line are the weighting factors that have to be applied to the I compensation sounds $i_1, i_2, \ldots, \ldots i_I$ in order to minimize the effect of the sound u_x. The statistically independent data that were assumed in connection with the derivation of $S_s(e^{j\theta})$ imply that the data transmitted by one sound do not contain information of the data transmitted by the other sounds. Accordingly, minimization of the effect of the data transmitted by one single sound is only carried out by considering this single sound and none else so that the equations described by equation (123) split into as many systems as sounds are used for transmitting information.

The coefficient matrix of all the U systems of equations are the same and can be represented as a block matrix.

$$A = \begin{pmatrix} A_{00} & A_{01} & \cdots & A_{0R-1} \\ A_{10} & A_{11} & \cdots & A_{1R-1} \\ \vdots & \vdots & \ddots & \vdots \\ A_{R-10} & A_{R-11} & \cdots & A_{R-1R-1} \end{pmatrix} \tag{124}$$

With the matrix A_{kl}, k, l = 0, 1, ..., R-1 defined as

$$A_{kl} = \int_0^{2\pi} W(e^{j\theta}) H_{\mathcal{I}}(e^{j\theta}) H_{\mathcal{I}}^{t}(e^{j\theta}) e^{-j\theta(l-k)N} \, d\theta \, . \tag{125}$$

The newly introduced value $\mathbf{H}_I(e^{j\theta})$ contains the Fourier transformed $\mathbf{H}_i(e^{j\theta})$ of the compensation sounds $h_i[n]$, $i \in K_I$.

$$H_{\mathcal{I}}^T(e^{j\theta}) = \left[H_{i_1}(e^{j\theta}) \; H_{i_2}(e^{j\theta}) \; \ldots \; H_{i_I}(e^{j\theta})\right] \qquad \{i_1, i_2, \ldots, i_I\} = \mathcal{K}_{\mathcal{I}} \tag{126}$$

The right side of the system of equations, which corresponds to $u = u_x$, may be written down as a block vector

$$d_{u_x}^T = \left[d_{u_x}^T[0] \; d_{u_x}^T[1] \; \ldots \; d_{u_x}^T[R-1]\right] \qquad \text{with}$$
$$d_{u_x}[r] = -\int_0^{2\pi} W(e^{j\theta}) H_{u_x}(e^{j\theta}) H_{\mathcal{I}}^{*}(e^{j\theta}) e^{j\theta r N} \, d\theta \quad \text{for} \quad r = 0, 1, \ldots, R-1 \, . \tag{127}$$

$H_{ux}(e^{j\theta})$ is the Fourier transformed of the base function $h_{ux}[n]$ of the sound u_x. With the equations (124) and (127), a system of equations is obtained for each $u_x \in U$